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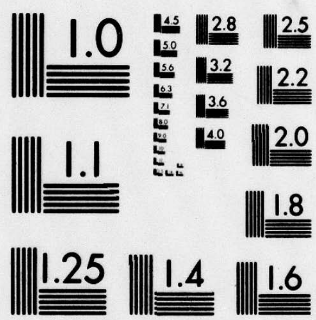

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Working Paper

Division of Research

GRADUATE SCHOOL of BUSINESS ADMINISTRATION

HARVARD UNIVERSITY

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Boston, Mass. 02163

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Harvard Univ~~ersity~~
Graduate School of Business Administration
335 Morgan Hall
Boston, Ma~~ssachusetts~~ 02163

This research was supported in part by the Naval Analysis Program, Office of Naval Research under Contract #NØØ014-78-C-Ø638, Task #NR-277-258.

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ADVANCES IN MULTIATTRIBUTE UTILITY THEORY

Peter H. Farquhar
Harvard University

Abstract

Several advances in multiattribute expected utility theory have emerged recently. Much of the existing theory deals with independence axioms on whole attributes and the corresponding utility decompositions. This paper reviews three alternate approaches for obtaining representations of multiattribute utility functions: (1) multivalent preference analysis, (2) approximation methods, and (3) spanning analysis. Unlike some utility decompositions, these approaches require the assessment of only single-attribute functions which makes implementation relatively simple. Only multivalent preference analysis and spanning analysis, however, provide axioms that can be empirically tested to justify a particular utility representation.

1. INTRODUCTION

The primary aim of utility analysis is the construction of mathematical representations of preferences that can aid in the evaluation of risky decisions. For several years, research in multiattribute expected utility theory has focused on the *decomposition approach*. This approach relies on various sets of independence axioms to prescribe how to divide the assessment of a multiattribute utility function into manageable components.

Though it is typically much easier to apply than a holistic analysis of preferences, the decomposition approach sometimes requires substantial effort from the decision maker. He must check the validity of different independence axioms and then assess the requisite conditional utility functions and scaling constants to obtain a utility decomposition. As the degree of interdependency among attributes increases, the effort needed to implement decomposition methods grows rapidly.

One can argue that the effort needed to represent some multiattribute preferences is not necessarily a reflection of inadequacies in the decomposition approach but rather an indication of the inherent complexity of particular decision problems. Nevertheless, there are many opportunities for improving the testing and assessment methods used in decomposing multiattribute utility functions.

This paper briefly reports on three emerging directions in multiattribute utility theory that appear to offer advantages over traditional decomposition methods whenever the attributes are interdependent. Section 2 reviews some independence axioms and utility representations to illustrate the decomposition approach, Section 3 examines recent research on multivalent preference structures, Section 4 looks at approximation methods for multiattribute utility assessment, and Section 5 considers new results from spanning analysis.

2. UTILITY DECOMPOSITIONS

Terminology

Let X denote the *outcome space* in a decision problem, and let P denote the space of all simple probability distributions (*lotteries*) over X. Let $\succ$ denote a *preference order* on P satisfying the von Neumann-Morgenstern axioms [13, 34]. Thus there exists a real function u on X, called a *utility function* for $\succ$ on P, such that for all p, q ε P, p $\succ$ q iff (if and only if) $\Sigma_{x \varepsilon X}\ p(x)u(x) > \Sigma_{x \varepsilon X}\ q(x)u(x)$.

For simplicity let $X = Y \times Z$, where Y and Z are *attribute* sets each containing at least two elements. P_Y denotes the set of all lotteries on Y. The (single-element) *conditional preference order* $\succ_z$ induced on P_Y by the preference order $\succ$ on P and a fixed element z ε Z is defined by

$$p_Y \succ_z q_Y \quad \text{iff} \quad (p_Y,\ z) \succ (q_Y,\ z), \tag{1}$$

where p_Y, q_Y ε P_Y.

Pollak [30], Keeney [22-26], Raiffa [31], and others have used the following independence axiom to derive various utility decompositions.

DEFINITION 1: Y is *utility independent* of Z, denoted Y(UI)Z, iff there exists a preference order $\succ_1$ on P_Y such that $\succ_z = \succ_1$ for all z ε Z.

Thus Y(UI)Z implies that preferences for lotteries on Y conditioned on a fixed element in Z do not depend on the fixed element. An analogous definition holds for Z(UI)Y.

Utility decomposition with two attributes

Since von Neumann-Morgenstern utility functions that preserve the same preference order are related by positive linear transformations [13], Y(UI)Z implies that for all y ε Y, z ε Z,

$$u(y,\ z) = \alpha(z) + \beta(z)u(y,\ z_0), \tag{2}$$

where z_0 is fixed arbitrarily in Z, and α and β are real functions on Z with $\beta > 0$.

Y is *essential* iff $\succ_z \neq \phi$ on P_Y for some $z \in Z$. In this case there exist y_0, $y_1 \in Y$ and a rescaling of u such that $u(y_0, z_0) = 0$ and $u(y_1, z_0) = 1$. Solving for α and β in (2) therefore yields [23]

$$u(y, z) = u(y_0, z) + [u(y_1, z) - u(y_0, z)]u(y, z_0), \tag{3}$$

for all $y \in Y$, $z \in Z$. Hence one conditional utility function on Y and two conditional utility functions on Z determine u when Y(UI)Z and Y is essential. (If Y is not essential, then $u(y, z) = u(y_0, z)$ trivially.)

The *partial* decomposition in (3) is replaced by a stronger result when Y and Z are mutually utility independent [22, 24].

THEOREM 1: Suppose that u is a von Neumann-Morgenstern utility function on $Y \times Z$, where Y and Z are both essential attributes. Let u be scaled so that $u(y_0, z_0) = 0$, $u(y_0, z_1) \neq 0$, and $u(y_1, z_0) \neq 0$. If Y is utility independent of Z and Z is utility independent of Y, then u has a *quasi-additive decomposition*,

$$u(y, z) = u(y, z_0) + u(y_0, z) + cu(y, z_0)u(y_0, z), \tag{4}$$

where c is a scaling constant defined by

$$c = [u(y_1, z_1) - u(y_0, z_1) - u(y_1, z_0)] \,/\, u(y_0, z_1)u(y_1, z_0). \tag{5}$$

Equation (4) yields the familiar *additive* decomposition $u(y, z) = u(y, z_0) + u(y_0, z)$ if $c = 0$ in (5) [12, 30]. On the other hand, (4) gives a *multiplicative* decomposition $u'(y, z) = u'(y, z_0) \cdot u'(y_0, z)$, where $u' \equiv 1 + cu$, if $c \neq 0$ in (5) [25, 30].

Decompositions with n attributes

Suppose the outcome space is $X = X_1 \times \ldots \times X_n$ for $n \geq 2$. Let $N \equiv \{1, \ldots, n\}$ be partitioned into nonempty sets I and $\bar{I}$, and let X_I denote the Cartesian product of the X_i for all $i \in I$. Then the definition of utility independence is easily extended with $Y \equiv X_I$ and $Z \equiv X_{\bar{I}}$.

Keeney [25, 26], Nahas [29], and others derive decompositions from various collections of utility independence assumptions on subsets of attributes. For example, if $X_I(UI)X_{\bar{I}}$ for all subsets $I \subset N$, then u has an additive or a multiplicative decomposition on n attributes. Other decompositions based on utility independence assumptions are discussed in [26, 29].

In some decision problems, utility independence does not hold because preferences for lotteries on Y indeed depend on the particular elements fixed in Z. Farquhar [4 - 7] describes a *fractional hypercube* methodology for generating different independence axioms and their corresponding multiattribute utility decompositions. One advantage of this methodology is that it provides a hierarchy of utility models ranging from the additive model to forms that represent increasingly complicated preference interdependencies among attributes. These interdependencies are reflected by interaction terms in the functional form of the utility decomposition. If the interaction terms are products of single-attribute functions (as in (4), for example), the decompositions are relatively easy to assess; on the other hand, the presence of nonseparable interaction terms complicates the assessment. Therefore applications have been limited primarily to the simpler types of fractional hypercube decompositions.

Detailed reviews of multiattribute utility decompositions and independence axioms are provided in [7, 19, 26].

Comments

A disadvantage in implementing the decomposition approach is that no independence axioms will be verified in some decision problems. This situation arises if for some reason it is undesirable to conduct independence tests or if certain axioms are tested and subsequently rejected. Each collection of independence axioms yields a particular utility decomposition, but in the absence of any empirical verification one must guess at the form of the utility function. One solution is to consider the approximation of an unknown or partially characterized utility function by different approximating forms. Another possibility is to develop axiomatic procedures which produce multi-attribute utility representations without the use of attribute independence axioms. These issues are addressed in the following sections.

The decomposition approach works well when preference interdependencies have simple forms [8]. For example, the additive and multiplicative representations have received wide application in utility analysis [26]. The research outlined in succeeding sections, however, focuses on methods for dealing with more complicated preference structures.

3. MULTIVALENT PREFERENCE ANALYSIS

Introduction

One approach in describing how preferences for lotteries on Y depend on elements in Z is to partition Z according to the distinct conditional preference orders induced on P_Y. Farquhar [9] introduces the following definition.

DEFINITION 2: The *multivalent preference structure of* Y *given* Z is defined by (Y, Ω_Z, [Z]) where for some nonempty index set $\mathcal{J}$,

(i) $\Omega_Z \equiv \{ \succ_j : \ j \in \mathcal{J} \}$ denotes a collection of distinct preference orders, called *base orders*, on P_Y; and

(ii) $[Z] \equiv \{\hat{Z}^j : \ j \in \mathcal{J} \}$ denotes a partition of Z into nonempty classes, called *orbitals*, such that $\succ_z = \succ_j$ for all $z \in \hat{Z}^j$ and $j \in \mathcal{J}$.

Note that two elements z' and z" in Z belong to the same orbital iff $\succ_{z'} = \succ_{z''}$ on P_Y.

Valence refers to the cardinality of [Z] in the preference structure (Y, Ω_Z, [Z]). At one extreme, the preference structure is *univalent* if $[Z] = \{Z\}$; Y is utility independent of Z in this case. At the other extreme, complete dependence of Y on Z occurs if [Z] consists of all single-element subsets of Z. Thus multivalent preference structures cover an entire spectrum of interdependencies between attributes.

Since Y is utility independent of the restriction of Z to $\hat{Z}$ for all orbitals $\hat{Z} \in [Z]$, a natural generalization of Definition 1 is

DEFINITION 3: Y is *multivalent utility independent* of [Z], denoted Y(UI)[Z], iff there exists a collection of base orders Ω_Z such that Y given Z has the multivalent preference structure (Y, Ω_Z, [Z]).

An analogous definition holds for Z(UI)[Y].

Multivalent utility representations

Farquhar [9] establishes the following representation theorem for multivalent preference structures involving two attributes (see [4, 28] also).

THEOREM 2: Let u be a von Neumann-Morgenstern utility function on the outcome space $Y \times Z$. Suppose Y is multivalent utility independent of [Z] and Z is multivalent utility independent of [Y]. Then there exist real functions α_1 and β_1 on Y with $\beta_1 > 0$, real functions α_2 and β_2 on Z with $\beta_2 > 0$, and constants $\hat{k}$ depending on the sets $\hat{Y} \times \hat{Z}$, where $\hat{Y} \in [Y]$ and $\hat{Z} \in [Z]$, such that u has one of the following *additive-multiplicative representations* for all $y \in \hat{Y}$ and $z \in \hat{Z}$:

$$u(y, z) = \alpha_1(y) + \alpha_2(z) + u(\hat{y}, \hat{z}), \tag{6a}$$

$$u(y, z) = \alpha_1(y) + \beta_1(y)u(\hat{y}, \hat{z}), \tag{6b}$$

$$u(y, z) = \alpha_2(z) + \beta_2(z)u(\hat{y}, \hat{z}), \tag{6c}$$

$$u(y, z) = \hat{k} + \beta_1(y)\beta_2(z)[u(\hat{y}, \hat{z}) - \hat{k}]. \tag{6d}$$

The assessment of the multivalent representations above is complicated by the number of conditional utility functions required to determine the α and β functions. However, vast simplification is possible with the following assumption.

DEFINITION 4: For y_0, $y_1 \in Y$, y_1 is *uniformly preferable* to y_0, denoted $y_1 \gg y_0$, iff $(y_1, z) \succ (y_0, z)$ for all $z \in Z$.

An analogous definition holds for $z_1 \gg z_0$. Farquhar [9] shows that when $y_1 \gg y_0$ and $z_1 \gg z_0$ in Theorem 2, u is completely specified in (6) by two conditional utility functions on each attribute, $u(y_0, z)$, $u(y_1, z)$, $u(y, z_0)$, $u(y, z_1)$, and the utilities assigned to $(\hat{y}, \hat{z}) \in \hat{Y} \times \hat{Z}$ for each $\hat{Y} \in [Y]$ and $\hat{Z} \in [Z]$.

Further results

The additive-multiplicative representations in Theorem 2 can be extended from two attributes to n attributes if certain uniform preferability assumptions are made. Other n-attribute representations can be derived from multivalent utility independence axioms [9, 10].

Instead of using conditional preference orders to determine the orbitals in [Z], one can obtain the same partition using equivalence relations.

DEFINITION 5: The relation *utility equivalence* (UE) on Z is defined by

$z'(UE)z''$ iff there exist constants a and b with $b > 0$ such that

$$u(y, z'') = a + bu(y, z') \quad \text{for all } y \in Y. \tag{7}$$

Since $z'(UE)z''$ iff $\succ_{z'} = \succ_{z''}$, the equivalence classes generated by (UE) are the orbitals in [Z] above.

Farquhar and Fishburn [10] define equivalence relations leading to multivalent forms of additive independence, utility independence, and fractional independence. These axioms generate a variety of multivalent representations of multiattribute utility functions.

Comments

The valence approach for assessing multiattribute utility functions partitions the elements of each attribute into equivalence classes, called orbitals, such that preference orders conditioned on the elements within each orbital are identical. Unlike decomposition methods that use independence axioms over whole attributes, the valence approach considers multivalent independence axioms for which particular independence relations hold on the restriction of each attribute to any of its orbitals. Since preference interdependencies among attributes are reflected primarily by the orbitals, attribute interactions are readily interpreted and the functional forms of the utility representations are kept simple. The valence approach not only subsumes decomposition methods, it also produces representations for preference structures not covered by previous methods.

The power of Theorem 2, for example, is that any two-attribute utility function can be represented by the additive-multiplicative forms in (6). As attribute interdependencies grow, however, the number of subspaces $\hat{Y} \times \hat{Z}$ increases accordingly. Similar, but less powerful, results hold for n-attribute utility functions. Further research is needed to refine the procedures for eliciting a decision maker's partition of the elements in each attribute, before applications of the valence approach can be judged. Areas of potential application of multivalent preference structures are suggested in [4, 9, 11, 28].

4. APPROXIMATION METHODS

The comments at the end of Section 2 indicate that in multiattribute utility analysis (1) independence axioms sometimes are not checked, and (2) decompositions involving only single-attribute functions are relatively easy to assess. These practical considerations have led to the study of *approximation methods* for multiattribute utility assessment.

For example, an additive function is used in many evaluation problems without any axiomatic justification. Considerable experience has shown, however, that additive models frequently give satisfactory results. In a review of interaction effects in multiattribute utility representations, Farquhar [8] describes behavioral research on (additive) approximations. Additional references are cited there. The presentation here covers recent mathematical investigations.

Approximations using interpolation

If Y is essential and utility independent of Z, then (3) holds. This equation can be rewritten as

$$u(y, z) = p(y)u(y_1, z) + (1 - p(y))u(y_0, z), \tag{8}$$

where $p(y) \equiv u(y, z_0)$, for all $y \in Y$ and $z \in Z$. Keeney [24, pp. 284-285] interprets (8) as interpolation between two conditional utility functions, $u(y_1, z)$ and $u(y_0, z)$.

One can begin with interpolation like (8) as a postulate and derive utility representations without resorting directly to independence axioms.

For example, Bell [1 - 3] defines a normalized conditional utility function $u(z|y) \equiv [u(y, z) - u(y, z_0)] / [u(y, z_1) - u(y, z_0)]$, where it is assumed $z_1 \gg z_0$. Then Z is *interpolated* by Y if there exists a real function θ on Y such that $\theta(y_0) = 0$, $\theta(y_1) = 1$, and

$$u(z|y) = \theta(y)u(z|y_1) + [1 - \theta(y)]u(z|y_0), \tag{9}$$

for all $y \in Y$ and $z \in Z$. Analogous definitions hold for $u(y|z)$ and Y interpolated by Z whenever $y_1 \gg y_0$. Bell [2] proves the following interpolation result.

THEOREM 3: Let u be a von Neumann-Morgenstern utility function on the outcome space $Y \times Z$. Suppose there exist y_0, $y_1 \in Y$ and z_0, $z_1 \in Z$ such that y_1 is uniformly preferred to y_0 and z_1 is uniformly preferred to z_0. Then Y is interpolated by Z and Z is interpolated by Y if and only if

$$\begin{aligned} u(y, z) = {} & a_0 + a_1 u(y|z_0) + a_2 u(z|y_0) - ku(y|z_0)u(z|y_0) \\ & + (k - a_1)u(y|z_0)u(z|y_1) + (k - a_2)u(y|z_1)u(z|y_0) \\ & + (a_{12} - k)u(y|z_1)u(z|y_1), \end{aligned} \tag{10}$$

where k is an arbitrary constant, $a_0 \equiv u(y_0, z_0)$, $a_1 \equiv u(y_1, z_0)$, $a_2 \equiv u(y_0, z_1)$, and $a_{12} \equiv u(y_1, z_1)$, for all $y \in Y$ and $z \in Z$.

The interpolation result in (10) requires the assessment of four single-attribute conditional utility functions and several scaling constants. This result is comparable to the multivalent representation in (6) with uniform preferability assumptions.

Tamura and Nakamura [32, 33] refer to (9) as *first-order convex dependence*. A natural extension of the interpolation idea in (9) is given by

<u>DEFINITION 6</u>: Z is *m-th order convex dependent* on Y, denoted $Z(CD_m)Y$, iff there exist distinct $y_0, y_1, \ldots, y_m \in Y$ and real functions $\theta_1, \ldots, \theta_m$ on Y such that $\theta_i(y_j) = \delta_{ij}$ (Kronecker's delta) for $i \in \{1, \ldots, m\}$ and $j \in \{0, 1, \ldots, m\}$ where

$$u(z|y) = [1 - \sum_{i=1}^{m} \theta_i(y)]u(z|y_0) + \sum_{j=1}^{m} \theta_j(y)u(z|y_j), \tag{11}$$

for all $y \in Y$ and $z \in Z$, where m is the smallest nonnegative integer for which (11) holds.

This definition leads to a grid of conditional utility functions from which utility approximations are determined by interpolation results similar to (10).

Nahas [29] describes an interpolation methodology based on *continuous cuts* which incorporates a major portion of the above research. The methodology considers approximations to u(y, z) of the form

$$f_1(y)u(y_1, z) + \ldots + f_m(y)u(y_m, z), \tag{12}$$

where $f_1, \ldots, f_m$ are real functions on Y and $y_1, \ldots, y_m$ are fixed in Y. Nahas focuses on properties like separability, risk, and sensitivity in developing utility approximations. Unfortunately, his work remains unpublished.

The interpolation results for two attributes have been extended to an arbitrary number of attributes, but these extensions are not addressed here [1, 3, 29, 32, 33].

Further results on approximation

A basic issue in approximation theory is the degree of error involved in using various approximating forms. Fishburn [16, 17] examines this issue for the uniform norm and approximations to a continuous u(y, z) of the form $f_1(y)g_1(z) + \ldots + f_m(y)g_m(z)$, where each function may involve one or more conditional utility functions or may be specified in a way that does not depend on u (cf., (11), (12)). Fishburn considers a number of elementary approximations based on additive, multiplicative, and other simple forms that yield exact results when certain independence axioms hold. He examines more general approximations using different types of linear interpolation and exact grid models. Many of these results relate to the approximation methods discussed above.

Comments

A major advantage of the utility approximation methods proposed by Fishburn [16, 17], Nahas [29], Bell [1 - 3], Tamura and Nakamura [31, 32], and others is that only single-attribute functions are used in the utility representations. Although such representations are comparatively easy to assess, the particular form of a representation depends on interpolation assumptions that are not directly testable. If the requisite assumptions cannot be empirically verified, the corresponding representations must be regarded as approximations to a multiattribute utility function. The goodness of the approximation, of course, is a central issue in using these methods. The approximation approach also includes preference regression, worth assessment, and other topics reviewed in [7, 8, 21, 26, 27].

5. SPANNING ANALYSIS

Let I denote a decision maker's indifference relation on lotteries over $Y \times Z$. The *conditional indifference relation* $I(y)$ induced on P_Z by the indifference relation I and a fixed element $y \in Y$ is defined by

$$p_Z \; I(y) \; q_Z \quad \text{iff} \quad (y, p_Z) \; I \; (y, q_Z), \tag{13}$$

where $p_Z, q_Z \in P_Z$.

Fishburn and Farquhar [18] introduce the following fundamental extension of utility independence.

DEFINITION 7: Z is *degree-n utility independent* of Y, denoted $Z(UI_n)Y$, iff Y contains a nonempty subset A with n elements, such that A is

(i) *independent*: $\bigcap_{y \in A \setminus \{y^*\}} I(y) \not\subseteq I(y^*)$ for all $y^* \in A$, and

(ii) *spanning*: $\bigcap_{y \in A} I(y) = \bigcap_{y \in Y} I(y)$.

The terminology above suggests certain analogies with the theory of linear vector spaces. Fishburn and Farquhar also give a procedure for determining independent spanning sets in Y.

Z is generalized utility independent of Y [15, 19, 20] iff $Z(UI_1)Y$. Finite-degree utility independence leads to several new utility representation theorems [18].

<u>THEOREM 4</u>: Let u be a von Neumann-Morgenstern utility function on the outcome space $Y \times Z$, where Y and Z are both essential attributes. Then Z is degree-n utility independent of Y for a positive integer n iff there exist real functions $f_1, \ldots, f_n$, a on Y and real functions $g_1, \ldots, g_n$ on Z such that

$$u(y, z) = f_1(y)g_1(z) + \ldots + f_n(y)g_n(z) + a(y), \tag{14}$$

for all $y \in Y$ and $z \in Z$, and such that the *sum-of-products representation* in (14) is valid for no positive integer smaller than n.

Theorem 4 implies that if $Z(UI_n)Y$, then $Y(UI_m)Z$ where $|n-m| \leq 1$. The sum-of-products representation in (14) thus leads to

<u>THEOREM 5</u>: Let u be a von Neumann-Morgenstern utility function on the outcome space $Y \times Z$, where Y and Z are both essential attributes. If Z is degree-n utility independent of Y with independent spanning set $\{y_1, \ldots, y_n\} \in Y$ and if Y is degree-m utility independent of Z with independent spanning set $\{z_1, \ldots, z_m\} \in Z$, then u has a *multiadditive representation* given by

$$u(y, z) = a_{00} + \sum_{i=1}^{n} a_{i0}u(y_i, z) + \sum_{j=1}^{m} a_{0j}u(y, z_j)$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{m} a_{ij}u(y_i, z)u(y, z_j), \qquad (15)$$

for all $y \in Y$ and $z \in Z$, where the a_{ij}'s are scaling constants.

The multiadditive representation in (15) requires n + m single-attribute conditional utility functions and at most (n+1)(m+1) scaling constants to assess u(y, z). This representation generalizes the quasi-additive decomposition in (4) and provides an axiomatic basis for interpolation results such as (10), (11), and others.

Comments

Spanning analysis appears to offer several advantages in multiattribute utility assessment. This approach provides (1) one functional representation for the entire outcome space (unlike multivalent preference analysis), (2) a set of testable axioms (unlike approximation methods), and (3) the assessment of only single-attribute conditional utility functions (unlike some utility decompositions). The utility representations are derived from axioms which use conditional indifference relations to construct so-called independent spanning subsets of each attribute.

It is too early to judge the usefulness of spanning analysis. Further research and applied studies should answer this question.

ACKNOWLEDGEMENT

This research was supported in part by the Office of Naval Research under Contract #N00014-78-C-0638, Task #NR-277-258.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER HBS 79-56	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Advances in Multiattribute Utility Theory		5. TYPE OF REPORT & PERIOD COVERED Technical Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Peter H. Farquhar		8. CONTRACT OR GRANT NUMBER(s) N00014-78-C-0638
9. PERFORMING ORGANIZATION NAME AND ADDRESS Harvard University Graduate School of Business Administration Boston, Massachusetts 02163		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61153N, RR-014-11-01, NR-277-258
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Analysis Program (Code 431) Office of Naval Research Arlington, Virginia 22217		12. REPORT DATE July 1979
		13. NUMBER OF PAGES 22
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) - - -		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

- - -

18. SUPPLEMENTARY NOTES

- - -

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Multiattribute utility theory	Approximation methods
Decision analysis	Indifference spanning analysis
Multivalent preference structures	Utility decompositions

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Several advances in multiattribute expected utility theory have emerged recently. Much of the existing theory deals with independence axioms on whole attributes and the corresponding utility decompositions. This report reviews three alternate approaches for obtaining representations of multiattribute utility functions: (1) multivalent preference analysis, (2) approximation methods, and (3) indifference spanning analysis. Unlike some utility decompositions, these approaches require the assessment of only single-attribute functions which makes implementation relatively simple.

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Professor F. Hutton Barron
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311 Summerfield Hall
University of Kansas
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Professor David E. Bell
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Morgan Hall 337
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Professor Samuel Bodily
Colgate Darden School of Business
Administration
University of Virginia
Carlottesville, VA 22903

Dr. Dean W. Boyd
Decision Focus, Inc.
1801 Page Mill Road
Palo Alto, CA 94304

Dr. Horace Brock
SRI International
Decision Analysis Group
333 Ravenswood Avenue
Menlo Park, CA 94025

Dr. Rex V. Brown
Decision Science Consortium, Inc.
Suite 421
7700 Leesburg Pike
Falls Church, VA 22043

Professor Jared L. Cohon
Dept. of Geography and
Environmental Engineering
Johns Hopkins University
Baltimore, MD 21218

Professor William W. Cooper
Graduate School of Business
Administration
Morgan Hall 16
Harvard University
Boston, MA 02163

Professor Norman C. Dalkey
School of Engineering and Applied
Science
University of California at Los
Angeles
Los Angeles, CA 90024

Dr. Robyn M. Dawes
Oregon Research Institute
P.O. Box 3196
Eugene, OR 97403

Decisions and Designs, Inc.
8400 Westpark Drive, Suite 600
P.O. Box 907
McLean, VA 22101

Professor Morris H. DeGroot
Department of Statistics
Carnegie-Mellon University
Pittsburgh, PA 15213

Professor James S. Dyer
Department of Management
College of Business Administration
University of Texas at Austin
Austin, TX 78712

Dr. Ward Edwards
Social Science Research Institute
University of Southern California
Los Angeles, CA 90007

Professor Hillel J. Einhorn
Graduate School of Business
University of Chicago
5836 Greenwood Avenue
Chicago, IL 60637

Professor Anthony Fiacco
Department of Operations Research
George Washington University
Washington, DC 20052

Professor Gregory W. Fischer
Social Science Department
Carnegie-Mellon University
Pittsburgh, PA 15213

Dr. Baruch Fischhoff
Decision Research
1201 Oak Street
Eugene, OR 97401

Dr. Peter C. Fishburn
Bell Telephone Laboratories, Inc.
600 Mountain Avenue
Murray Hill, NJ 07974

Dr. Barbara Goodman
Decision Analysis Co.
8020 Lincoln
Huntington Woods, MI 48070

Professor Paul E. Green
Department of Marketing
Wharton School of Management
University of Pennsylvania
Philadelphia, PA 19174

Dr. Henry Halff
Code 458
Office of Naval Research
800 N. Quincy Street
Arlington, VA 22217

Dr. Kenneth R. Hammond
Institute of Behavioral Science
University of Colorado
Boulder, CO 80302

Professor John R. Hauser
Department of Marketing
Graduate School of Management
Northwestern University
Evanston, IL 60201

Dr. Tom Hemming
Wibons väg 12 9tr
S-171 60 Solna
SWEDEN

Professor Arthur P. Hurter, Jr.
Dept. of Industrial Eng. and
Management Science
Technological Institute
Northwestern University
Evanston, IL 60201

Professor Y. C. Ho
Engineering and Applied Mathematics
Pierce Hall
Harvard University
Cambridge, MA 02138

Professor Charles A. Holloway
Graduate School of Business
Stanford University
Stanford, CA 94305

Professor Ronald A. Howard
Department of Engineering-
Economic Systems
School of Engineering
Stanford University
Stanford, CA 94305

Professor George P. Huber
Graduate School of Business
University of Wisconsin at Madison
1155 Observatory Drive
Madison, WI 53706

Professor Patrick Humphreys
Department of Psychology
Brunel University
Kingston Lane
Uxbridge, Middlesex UB8 3PH
GREAT BRITAIN

Dr. Edgar M. Johnson
Organizations and Systems Research Laboratory
U.S. Army Research Institute for the Behavioral and Social Sciences
5001 Eisenhower Avenue
Alexandria, VA 22333

Professor Gordon M. Kaufman
Sloan School of Management
Massachusetts Institute of Technology
Cambridge, MA 02139

Dr. Donald L. Keefer
Gulf Management Sciences Group
Gulf Science and Technology Company
P.O. Box 1166
Pittsburgh, PA 15230

Dr. Thomas W. Keelin
Decision Focus, Inc.
1801 Page Mill Road
Palo Alto, CA 94304

Dr. Ralph L. Keeney
Woodward-Clyde Consultants
Three Embarcardero Center, Suite 700
San Francisco, CA 94111

Dr. Craig W. Kirkwood
Woodward-Clyde Consultants
Three Embarcardero Center, Suite 700
San Francisco, CA 94111

Professor Jonathan Kornbluth
Department of Decision Sciences
Wharton School of Management
University of Pennsylvania
Philadelphia, PA 19174

Jeffrey P. Krischer, Ph.D.
Chief, Health Services Research
Veterans Administration Hospital
Gainesville, FL 32602

Professor Irving H. LaValle
School of Business Administration
Tulane University
New Orleans, LA 70118

Professor Sang M. Lee
Department of Management
University of Nebraska at Lincoln
Lincoln, NB 68588

Dr. Patrick Leung
Social Science Research Institute
University of Southern California
Los Angeles, CA 90007

Dr. Roger Levien
International Institute of Applied Systems Analysis
Schloss Laxenburg
A-2361 Laxenburg
AUSTRIA

Dr. Sarah Lichtenstein
Decision Research
1201 Oak Street
Eugene, OR 97401

Professor William F. Lucas
School of Operations Research & Industrial Engineering
Upson Hall
Cornell University
Ithaca, NY 14853

Professor R. Duncan Luce
Department of Psychology and Social Relations
William James Hall, Room 930
Harvard University
Cambridge, MA 02138

Professor Kenneth R. MacCrimmon
Faculty of Commerce and Business Administration
University of British Columbia
Vancouver, BC V6T1W5
CANADA

Dr. James E. Matheson
Decision Analysis Group
SRI International
333 Ravenwood Avenue
Menlo Park, CA 94025

Professor Richard F. Meyer
Graduate School of Business Administration
Morgan Hall 315
Harvard University
Boston, MA 02163

Dr. Peter A. Morris
Xerox Palo Alto Research Center
3333 Coyote Hill Road
Palo Alto, CA 94304

Professor Joel N. Morse
Dept. of Business Administration
University of Delaware
Newark DE 19711

Professor V. M. Ozernoi
Institute of Control Sciences
Academy of Sciences of the USSR
Moscow, USSR

Professor Howard Raiffa
Graduate School of Business Administration
Morgan Hall 302
Harvard University
Boston, MA 02163

Professor Vithala R. Rao
Graduate School of Business and Public Administration
Malott Hall
Cornell University
Ithaca, NY 14853

Professor Scott Richard
Graduate School of Industrial Management
Carnegie-Mellon University
Pittsburgh, PA 15213

Professor Patrick Rivett
Operational Research Department
University of Sussex
Brighton BNI9RF,
GREAT BRITAIN

Professor Fred S. Roberts
Department of Mathematics
Rutgers University
New Brunswick, NJ 08903

Professor Andrew P. Sage
Office of the Dean
School of Engineering and Applied Science
University of Virginia
Charlottesville, VA 22901

Professor Rakesh K. Sarin
Graduate School of Management
University of California at Los Angeles
Los Angeles, CA 90024

Professor Martin Shubik
Department of Economics
Yale University
Box 2125, Yale Station
New Haven, CT 06520

Dr. Paul Slovic
Decision Research
1201 Oak Street
Eugene, OR 97401

Dr. Richard D. Smallwood
Applied Decision Analysis
3000 Sand Hill Road
Menlo Park, CA 94025

Professor Richard Soland
Department of Operations Research
School of Engineering and Applied Science
George Washington University
Washington, DC 20052

Professor Ralph E. Steuer
College of Business and Economics
University of Kentucky
Lexington, KY 40506

Professor Hiroyuki Tamura
Department of Precision Engineering
Osaka University
Yamada-kami, Suita, Osaka 565
JAPAN

Professor Robert M. Thrall
Dept. of Mathematical Sciences
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Houston, TX 77001

Dr. Martin Tolcott
Code 455
Office of Naval Research
800 N. Quincy Street
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Professor Amos Tversky
Faculty of Social Science
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Professor Thomas S. Wallsten
L.L. Thurstone Psychometric
Laboratory
University of North Carolina
Chapel Hill, NC 27514

Professor Donald A. Wehrung
Faculty of Commerce and Business
Administration
University of British Columbia
Vancouver, BC VGT1W5
CANADA

Professor Robert B. Wilson
Department of Decision Sciences
Graduate School of Business
Administration
Stanford University
Stanford, CA 94305

Professor Robert L. Winkler
Graduate School of Business
Indiana University
Bloomington, IN 47401

Dr. Detlof von Winterfeldt
Social Science Research Institute
University of Southern California
Los Angeles, CA 90007

Professor P. L. Yu
307 Summerfield Hall
School of Business
University of Kansas
Lawrence, KS 66045

Professor Milan Zeleny
Graduate School of Business
Uris Hall
Columbia University
New York, NY 10027

Professor Stanley Zionts
School of Management
State University of New York
Buffalo, NY 14214